



## Objectives

## What are expression templates?

- an example of using templates for runtime-efficient computations
  - an example of modern template programming techniques
    - template meta-programming
    - generative programming

## Where it all started ...

- Erwin Unruh's prime number program ...
  - does not compile, but
  - calculates the prime numbers at compile-time and
  - emits them in error messages.
- works via recursive template evaluation
- useful for
  - evaluation of expressions (vector dot product, matrix operations)
  - calculation of constants (square root of N, prime numbers)
  - evaluation of logical expression (more readable STL functors)

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(3)

## Agenda

- compile-time computation of constant values
  - factorial
  - square root
- compile-time evaluation of expressions
  - dot product
  - arithmetic expression
- more examples of modern template programming
  - compile-time polymorphism
  - policy classes
  - template meta-programming

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(4)

## Runtime Computation of Factorial

The factorial of  $n$  is  $1 \cdot 2 \cdots \cdot (n-1) \cdot n$ , and  
the factorial of  $0$  is  $1$ .

A recursive factorial function:

```
int factorial (int n)
{ return (n==0) ? 1: n*factorial (n-1); }
```

```
cout << factorial (4) << endl ;
```

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(5)

## Compile-Time Counterpart

A class for compile time computation of the factorial:

```
template <int n>
struct Factorial {
    enum { RET = Factorial <n-1>::RET * n };
};

template <>
struct Factorial <0> {
    enum { RET = 1 };
};
```

```
cout << Factorial <4>::RET << endl ;
```

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(6)

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(7)

## Compile-Time Computation of Square Root

Given  $N$ , compute a compile-time table size `ceil(sqrt(N))`.

Example: `int table[Root<10>::root];`

Beginning of recursion:

```
template <size_t Size, size_t Low=1, size_t High=Size>
struct Root;
```

`Root<10> => Root<10, 1, 10>`

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(8)

## The Recursion

Recursive definition of Root::root:

```
template <size_t Size, size_t Low, size_t High>
struct Root {
    static const size_t root =
        Root<Size, (down?Low:mean+1), (down?mean:High)>
        ::root;

    static const size_t mean = (Low+High)/2;
    static const bool down = ((mean*mean)>=Size);
};
```

```
Root<10, 1, 10> => Root<10, 1, 5>
mean = (1+10)/2 = 5
down = (5*5>=10) = true
```

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(9)

## End of the Recursion

```
Root<10, 1, 5> => Root<10, 4, 5>
mean = (1+5)/2 = 3
down = (3*3>=10) = false
```

```
Root<10, 4, 5> => Root<10, 4, 4>
mean = (4+5)/2 = 4
down = (4*4>=10) = true
```

End of recursion:

```
template <size_t Size, size_t Mid>
struct Root<Size, Mid, Mid>
{ static const size_t root = Mid; };
```

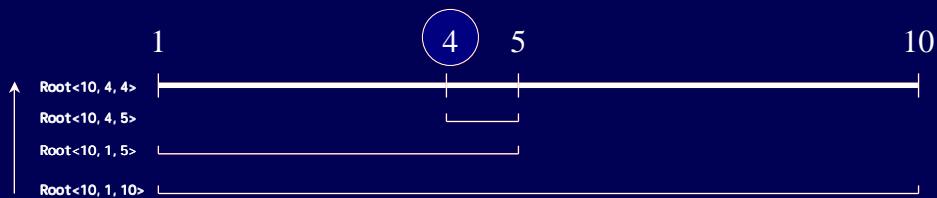
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(10)

## Result

Given  $N$ , compute a compile-time table size  $\lceil \sqrt{N} \rceil$ !

Example: `int table[Root<10> : root];`



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(11)

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(12)

## Goal

- efficient computation of arithmetic expressions
- example:
  - dot product of 2 vectors of dimension N

```
int a[4] = {1, 100, 0, -1};  
int b[4] = {2, 2, 2, 2};  
dot<4>(a, b);
```

- more generally:
  - arithmetic operations on multi-dimensional matrices

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(13)

## More Dynamic Use

- compile-time computation of integrals

$$\int_{1.0}^{5.0} \frac{x}{1+x}$$

```
template <class ExprT>  
double integrate (ExprT e, double from, double to, size_t n)  
{  
    double sum=0, step=(to-from)/n;  
    for (double i=from+step/2; i<to; i+=step)  
        sum+=e.eval(i);  
    return step*sum;  
}  
  
Identity x;  
cout << integrate (x/(1.0+x), 1.0, 5.0, 10) << endl;
```

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(14)

## Gauss Distribution

```
doubl e sigma=2.0, mean=5.0;
const doubl e Pi = 3.141593;
cout << integrate(
    1.0/(sqrt(2*Pi)*sigma) * exp(sqr(x-mean)/(-2*sigma*sigma))
    , 2.0, 10.0, 100) << endl ;
```

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(15)

## STL Predicates

```
list<int> l;
identity x;
count_if(l.begin(), l.end(), x >= 0 && x <= 100);
```

... provided the logical and comparison operators are overloaded to yield expressions that evaluate to Boolean values

- another example of expression templates
  - improve readability
  - at no additional runtime cost

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(16)

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(17)

## Understanding Expression Templates

- try to understand template approach as a alternative to the classic OO approach
- expressions evaluation is an example of patterns in GOF
  - composite
  - interpreter

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(18)

## The Composite Pattern

The Composite pattern provides a way to represent a part-whole relationship where the client can ignore the difference between individual objects and compositions of objects.

- The *leaf* (terminal) defines the behavior for primitive objects in the composition.
- The *composite* (non-terminal) defines the behavior for components consisting of leaves.

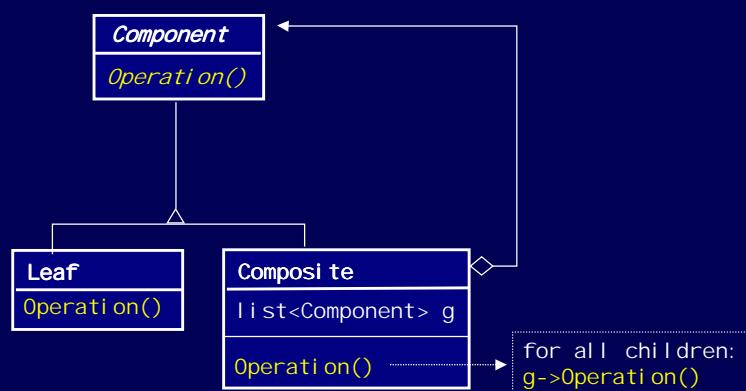
Examples:

- syntax trees, expression evaluation
- aggregations and recursive structures and algorithms

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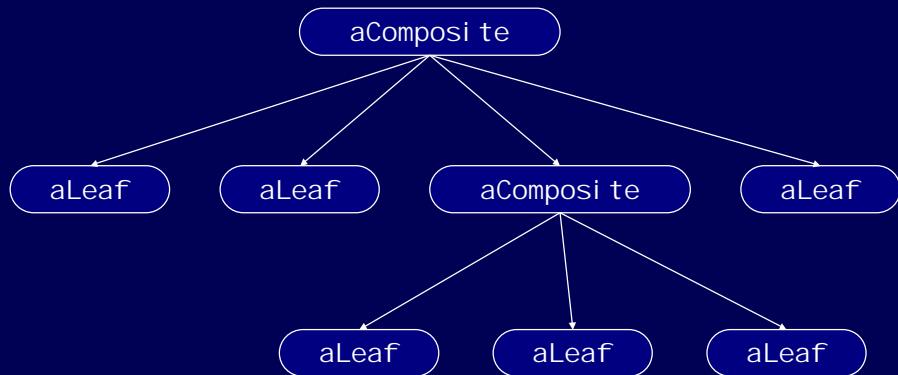
## The Composite Pattern



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(20)

## A Typical Composite Structure



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(21)

## An Example - Vector Dot Product

Component:

- dot product of vectors of dimension N

Leaf:

- simple product of two numerical values, i.e. the dot product of a vector of dimension 1

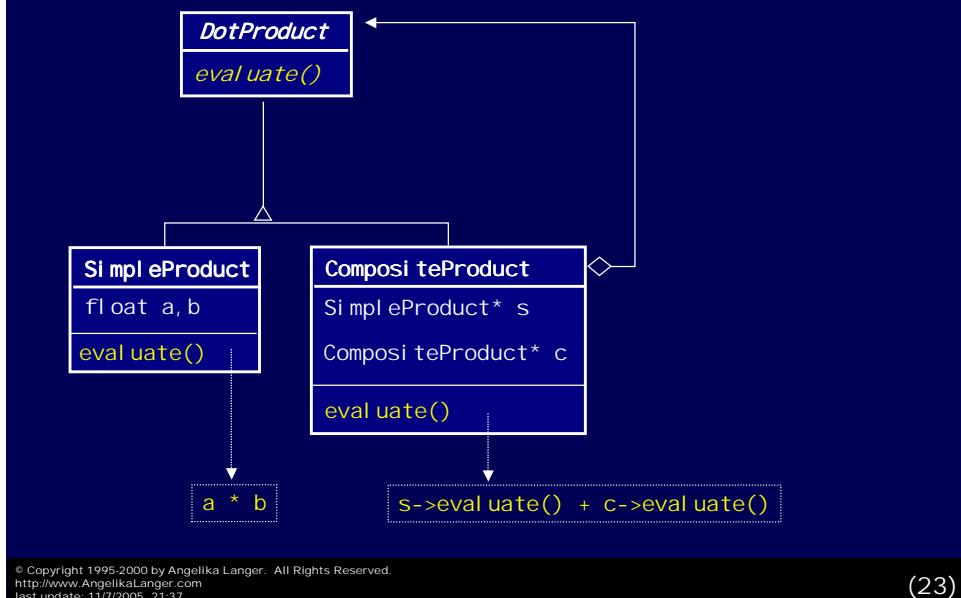
Composite:

- consists of a leaf (dimension 1) and a composite (dimension N-1)
- calculation of the sum of the leaf's value and the composite's value

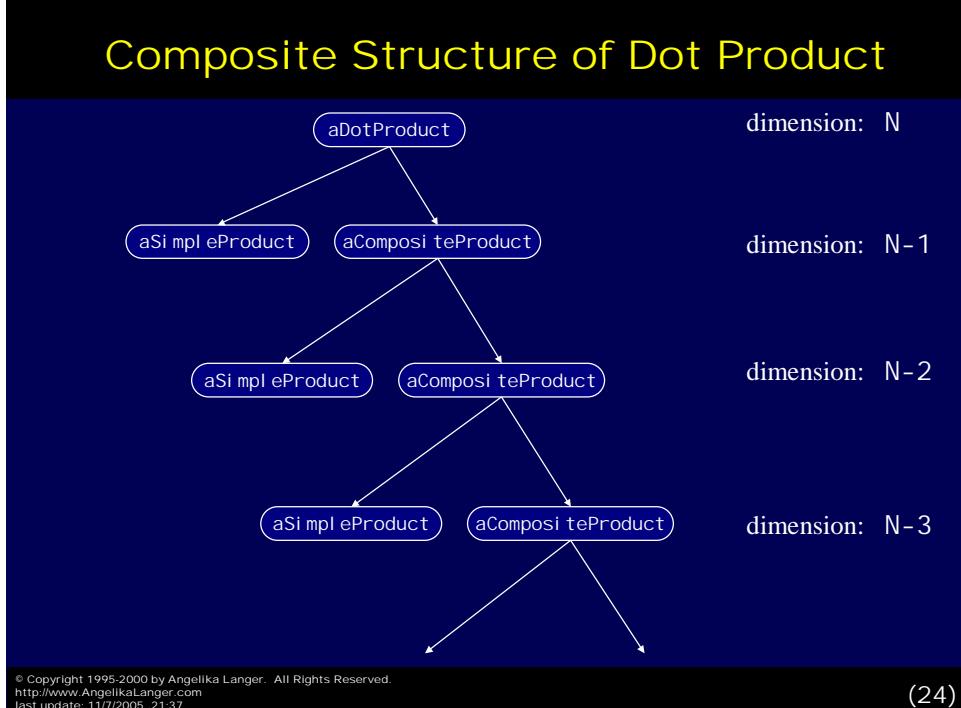
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(22)

## OO Implementation à la GOF



## Composite Structure of Dot Product



## The Component

The base class that defines the interface common to leaves and composites:

```
template <class T>
class DotProduct {
public:
    virtual ~DotProduct () {}
    virtual T eval () = 0;
};
```

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(25)

## The Composite

```
template <class T>
class CompositeDotProduct : public DotProduct<T> {
public:
    CompositeDotProduct (T* a, T* b, size_t dim)
        : s(new SimpleDotProduct<T>(a, b))
        , c((dim==1)?0: new CompositeDotProduct<T>(a+1, b+1, dim-1))
    {}
    virtual ~CompositeDotProduct () { delete c; delete s; }
    virtual T eval ()
    { return (s->eval () + ((c)?c->eval ():0)); }
protected:
    SimpleDotProduct<T>* s;
    CompositeDotProduct<T> * c;
};
```

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(26)

## The Leaf

```
template <class T>
class SimpleDotProduct : public DotProduct <T> {
public:
    SimpleDotProduct (T* a, T* b) : v1(a), v2(b) {}
    virtual T eval () { return (*v1)*(*v2); }
private:
    T* v1; T* v2;
};
```

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(27)

## The Client

The code that initiates the recursive evaluation of the composite structure:

```
template <class T> T dot(T* a, T* b, size_t dim)
{ return (dim==1)
    ? SimpleDotProduct<T>(a, b).eval ()
    : CompositeDotProduct<T>(a, b, dim).eval ();
}

int a[4] = {1, 100, 0, -1};
int b[4] = {2, 2, 2, 2};
dot(a, b, 4);
```

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(28)

## A Simplified OO Implementation

Eliminate the representation of the composite and the leaf object as data members.

- Instead of passing information to the constructor and memorizing them for subsequent evaluation,
- pass them to the directly to the evaluation function.

```
Si mpleDotProduct (T* a, T* b) :v1(a), v2(b) {}  
vi rtual T eval () { return (*v1)*(*v2); }
```

becomes

```
T eval (T* a, T* b, size_t dim) { return (*a)*(*b); }
```

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(29)

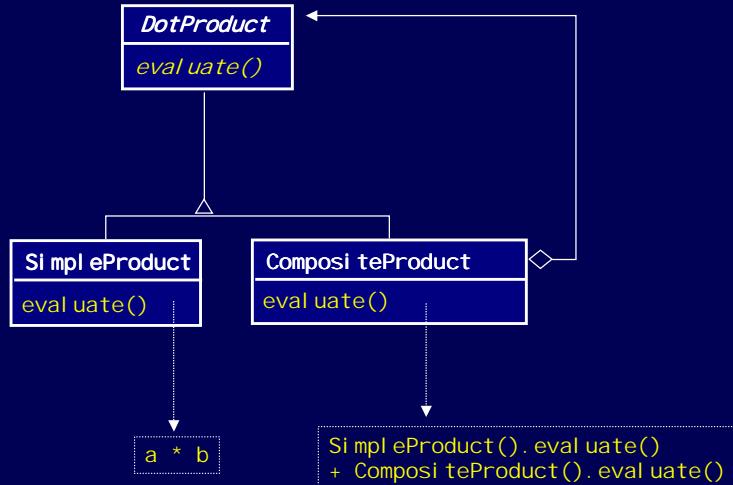
## A Simplified OO Implementation

```
templat e <cl ass T>  
cl ass Composit eDotProduct : publ ic DotProduct <T> {  
publ ic:  
    vi rtual T eval (T* a, T* b, size_t dim)  
    { return Si mpleDotProduct<T>(). eval (a, b, dim)  
        + ((dim==1) ? 0  
            : Composit eDotProduct<T>(). eval (a+1, b+1, dim-1));  
    }  
};  
  
templat e <cl ass T>  
cl ass Si mpleDotProduct : publ ic DotProduct <T> {  
publ ic:  
    vi rtual T eval (T* a, T* b, size_t dim)  
    { return (*a)*(*b); }  
};
```

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(30)

## A Simplified OO Implementation



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(31)

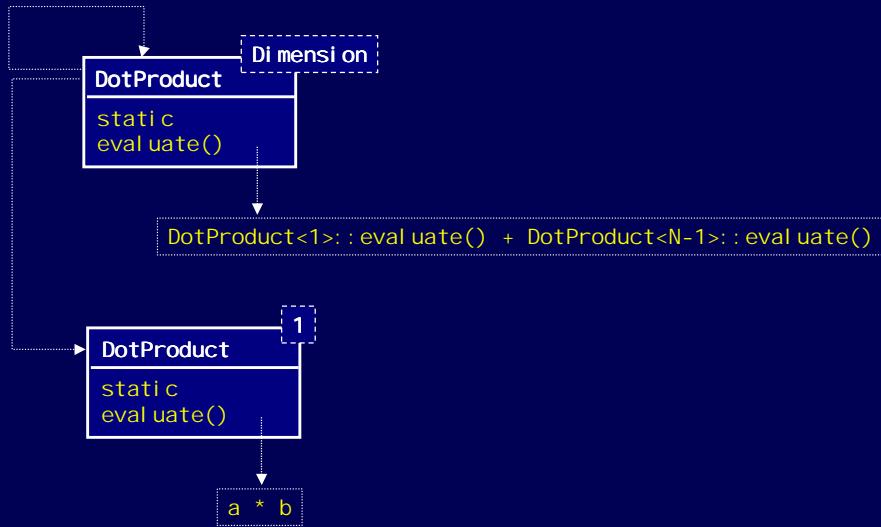
## Implementation Using Templates

- The template solution does not need a base class.
  - Eliminate the Component base class.
- Implement the Composite as a class template using structural information as template arguments.
  - The dimension of the vector becomes a non-type template argument of the Composite.
- Implement the Leaf as a specialization of the Composite.
  - The SimpleDotProduct is a specialization of the CompositeDotProduct for dimension N = 1.
- Instead of run time recursion use compile time recursion.
  - Replace the recursive invocation of the virtual evaluation function by recursive template instantiation of a static evaluation function.

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(32)

## Implementation Using Templates



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(33)

## The Composite

```
template <size_t N, class T>
class DotProduct {
public:
    static T eval(T* a, T* b)
    { return DotProduct<1, T>::eval(a, b)
      + DotProduct<N-1, T>::eval(a+1, b+1);
    }
};
```

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(34)

## The Leaf

A specialization of the Composite class template for dimension N = 1:

```
template <class T>
class DotProduct<1, T> {
public:
    static T eval (T* a, T* b)
    { return (*a)*(*b); }
};
```

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(35)

## The Client

```
template <size_t N, class T>
T dot(T* a, T* b)
{ return DotProduct<N, T>::eval (a, b); }

int a[4] = {1, 100, 0, -1};
int b[4] = {2, 2, 2, 2};
dot<4>(a, b);
```

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(36)

## Composite and Interpreter

The dot product example is a degenerated form of a Composite because

- every Composite consists of exactly one Leaf and one Composite, and
- there is only one type of Leaf and one type of Composite.

The Interpreter pattern is a related pattern.

- The syntax tree in the Interpreter pattern is a Composite.

Let us discuss alternative implementations of the Interpreter pattern.

## Agenda

- compile-time computation of constant values
  - factorial
  - square root
- **compile-time evaluation of expressions**
  - dot product
  - arithmetic expression
- more examples of modern template programming
  - compile-time polymorphism
  - policy classes
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## The Interpreter Pattern

The Interpreter pattern provides a way to represent a language in form of an abstract syntax tree and an interpreter that uses the syntax tree to interpret language constructs.

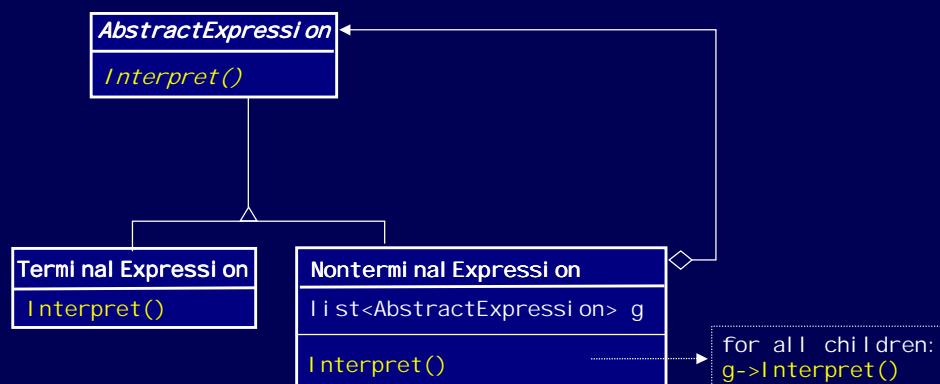
The part-whole relationship of the Composite pattern corresponds to the relationship of an expression and its subexpressions in the Interpreter pattern.

- The *leaf* is a terminal expression.
- The *composite* is a non-terminal expression.
- *Evaluation* of the components is interpretation of the syntax tree and its expressions.

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(39)

## The Interpreter Pattern



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(40)

## Example: Arithmetic Expressions

SyntaxTree:

- the representation of an arithmetic expression such as  
 $(a+1)*c$  or  $\log(\text{abs}(x-N))$

Terminal:

- a numerical literal such as a constant of type double
- a reference to a variable of type double;  
its value might change between interpretations of the expression

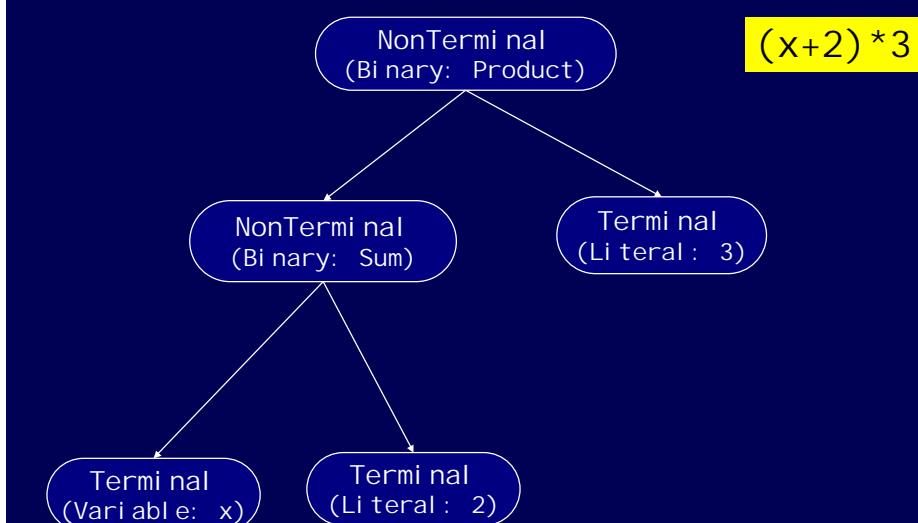
NonTerminal:

- unary and binary expression consisting of one or two subexpressions;
- the expressions have different semantics such as +, -, \*, /, ++, --, exp, log, sqrt

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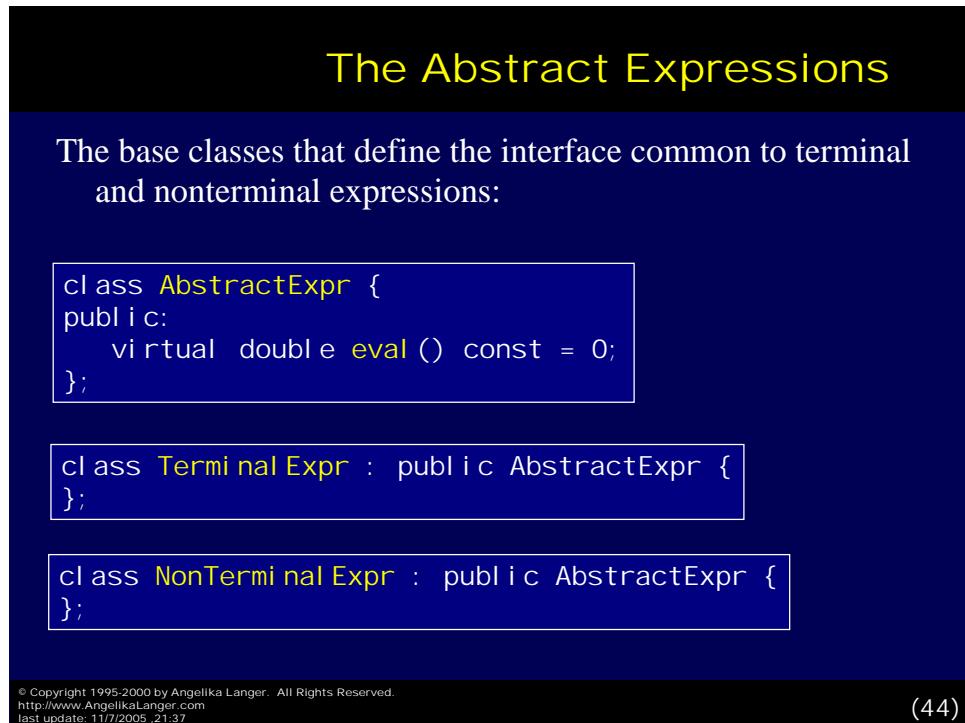
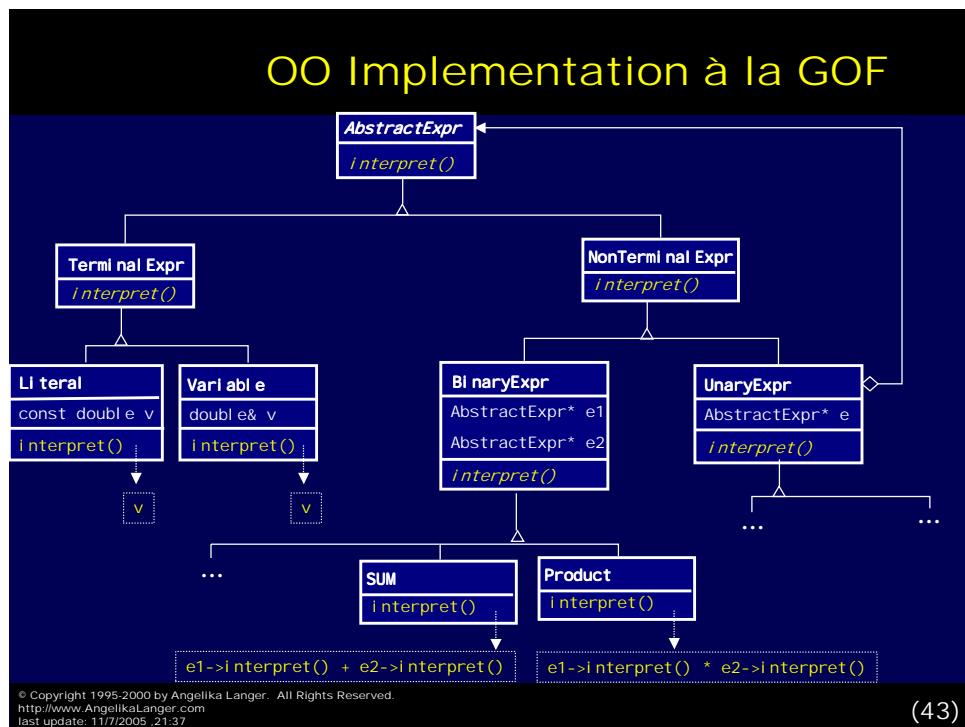
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## A Sample Syntax Tree



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(42)



## The Terminal Expressions

```
class Literal : public TerminalExpr {  
public:  
    Literal(double v) : _val(v) {}  
    double eval() const { return _val; }  
private:  
    const double _val;  
};
```

```
class Variable : public TerminalExpr {  
public:  
    Variable(double v) : _val(v) {}  
    double eval() const { return _val; }  
private:  
    double _val;  
};
```

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(45)

## The Non-Terminal Expressions

```
class BinaryExpr : public NonTerminalExpr {  
protected:  
    BinaryExpr(const AbstractExpr* e1, const AbstractExpr* e2)  
        : _expr1(e1), _expr2(e2) {}  
    virtual ~BinaryExpr ()  
    { delete const_cast<AbstractExpr*>(_expr1);  
     delete const_cast<AbstractExpr*>(_expr2);  
    }  
    const AbstractExpr* _expr1;  
    const AbstractExpr* _expr2;  
};
```

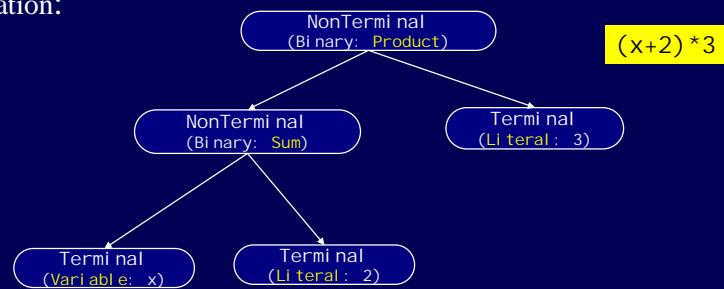
```
class Sum : public BinaryExpr {  
public:  
    Sum(const AbstractExpr* e1, const AbstractExpr* e2)  
        : Bi nExpr(e1, e2) {}  
    double eval() const  
    { return _expr1->eval() + _expr2->eval(); }  
};
```

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(46)

## The Client

The code that creates the syntax tree and initiates its recursive interpretation:



```
double x;  
Product expr(new Sum(new Variable(x), new Literal(2)),  
            new Literal(3));  
cout << expr.eval() << endl;
```

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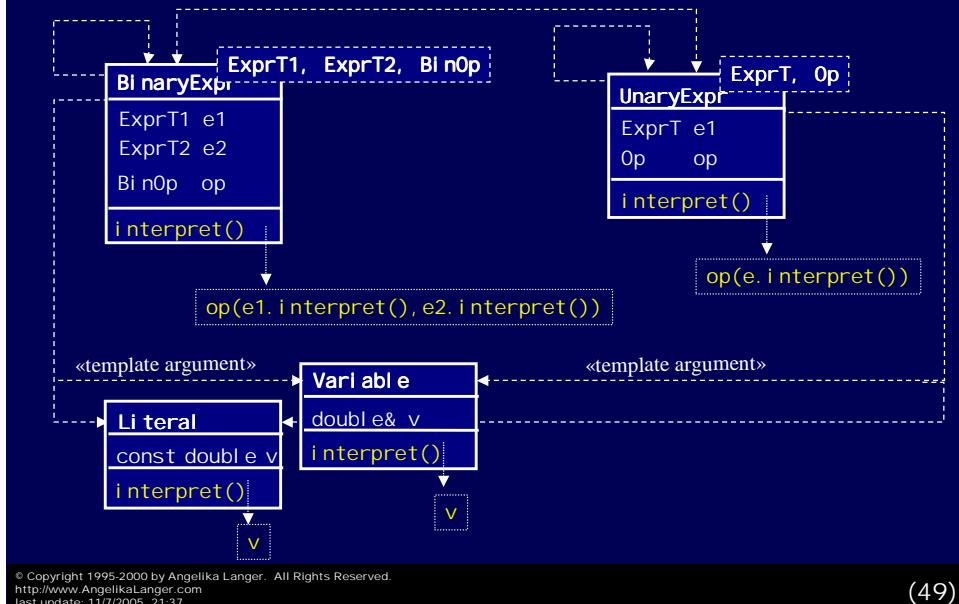
## Implementation Using Templates

- The template solution does not need a base class.
  - Eliminate all abstract base classes.
- Implement the NonTerminalExpression as a class template using structural information as template arguments.
  - The types of the subexpressions are type template arguments of the NonTerminalExpression.
- Parameterize the NonTerminalExpression with the type of operation.
  - The actual operation (+,-,\*,/,++,--,abs,exp,log) is stored as a function object and its type is a template argument.
- Implement the TerminalExpressions as normal classes.
- Instead of run time recursion use compile time recursion.
  - Replace the recursive invocation of the virtual evaluation function by recursive template instantiation.

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(48)

## Implementation Using Templates



(49)

## The Terminal Expressions

```
class L i teral {
public:
    L i teral (const doubl e v) : _val (v) {}
    doubl e eval () const { return _val; }
private:
    const doubl e _val;
};
```

```
class Vari abl e {
public:
    Vari abl e(doubl e& v) : _val (v) {}
    doubl e eval () const { return _val; }
private:
    doubl e& _val;
};
```

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(50)

## The Non-Terminal Expressions

```
template <class ExprT1, class ExprT2, class Bi nOp>
class Bi naryExpr {
public:
    Bi naryExpr(ExprT1 e1, ExprT2 e2, Bi nOp op=Bi nOp())
        : _expr1(e1), _expr2(e2), _op(op) {}
    double eval() const
    { return _op(_expr1.eval(), _expr2.eval()); }
private:
    ExprT1 _expr1;
    ExprT2 _expr2;
    Bi nOp _op;
};
```

As operations we can use the pre-defined STL function objects plus, minus, multiplies, divides, etc. or define our own functions objects as needed.

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(51)

## Creator Functions

A binary expression representing a sum would be of type  
**Bi nExpr< ExprT1, ExprT2, plus<double> >**

For convenience, define creator functions that take advantage of automatic template argument deduction for function templates:

```
template <class ExprT1, class ExprT2>
Bi naryExpr<ExprT1, ExprT2, plus<double> >
makeSum(ExprT1 e1, ExprT2 e2)
{ return Bi naryExpr<ExprT1, ExprT2, plus<double> >(e1, e2); }
```

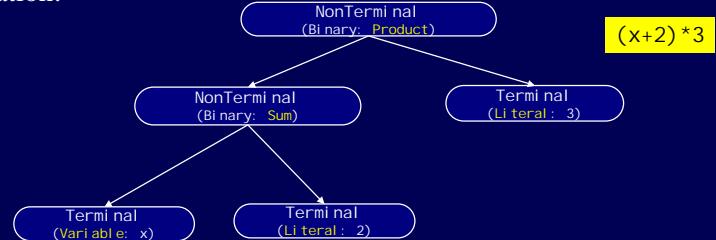
```
template <class ExprT1, class ExprT2>
Bi naryExpr<ExprT1, ExprT2, multiplies<double> >
makeProd(ExprT1 e1, ExprT2 e2)
{ return Bi naryExpr<ExprT1, ExprT2, multiplies<double> >(e1, e2); }
```

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(52)

## The Client

The code that creates the syntax tree and initiates its recursive interpretation:



```
double x;
BinaryExpr< BinaryExpr < Variable, Literal, plus<double> >,
           Literal,
           multiplies<double> >
expr = makeProd (makeSum (Variable(x), Literal(2)),
                  Literal(3));
cout << expr.eval () << endl;
```

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(53)

## Operator Overloading

Often the type of the expression is not even needed:

```
double x;
cout << makeProd(makeSum(Variable(x), Literal(2)), Literal(3)).eval ()
<< endl;
```

Take advantage of operator overloading in C++ and define the creator functions as overloaded operators.

```
Variable v(x);
cout << eval ((v+2)*3.0) << endl;
```

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(54)

## First attempt

Define the creator functions as overloaded operators:

```
template <class ExprT1, class ExprT2>
BinaryExpr <ExprT1, ExprT2, plus<double> >
operator+(ExprT1 e1, ExprT2 e2)
{ return BinaryExpr <ExprT1, ExprT2, plus<double> >(e1, e2); }
```

An expression such as **x+2** would evaluate to

**BinaryExpr <double, int, plus<double> >(x, 2)**

which is not exactly what we need. From just the type the compiler cannot distinguish between a literal and a variable of type double. We need a binary expression that stores a Variable and a Literal expression internally.

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(55)

## Requirement

Define variables as Variable expressions:

```
double x; Variable v(x);
cout << eval ((v+2)*3.0) << endl;
```

The compiler can now deduce:

**BinaryExpr <Variable, int, plus<double> >(v, 2)**

Inside the binary expression the variable would be stored as an expression of type Variable that refers to the variable x of type double.

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(56)

## Solution

Inside the binary expression store constants of any numerical type as Literal expressions.

For this purpose define for each expression type and all numerical types what their corresponding expression type is:

```
template <class ExprT> struct exprTraits
{ typedef ExprT expr_type; };

template <> struct exprTraits<double>
{ typedef Literal expr_type; };

template <> struct exprTraits<int>
{ typedef Literal expr_type; };

...
```

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(57)

## Modify the BinaryExpression Class

Inside the binary expression convert expressions to their expression type as defined in the expression traits:

```
template <class ExprT1, class ExprT2, class BiOp>
class BinaryExpr {
public:
    BinaryExpr(ExprT1 e1, ExprT2 e2, BiOp op=BiOp())
        : _expr1(e1), _expr2(e2), _op(op) {}
    double eval() const
    { return _op(_expr1.eval(), _expr2.eval()); }
private:
    exprTraits<ExprT1>::expr_type _expr1;
    exprTraits<ExprT2>::expr_type _expr2;
    BiOp _op;
};
```

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(58)

## Fine Tuning

Define a helper function:

```
template <class ExprT>
double eval(ExprT e) { return e.eval(); }
```

In the example

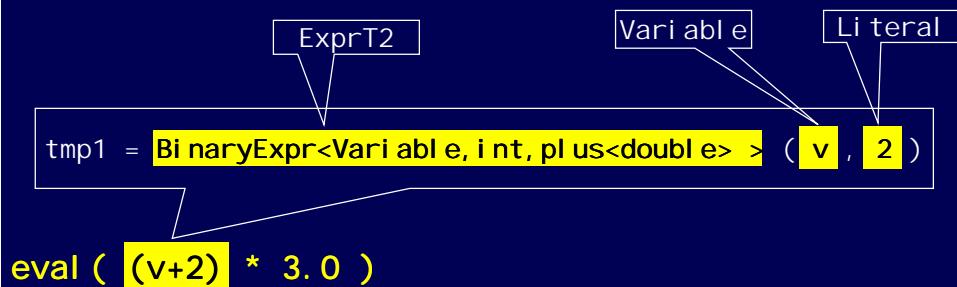
```
double x; Variable v(x);
cout << eval((v+2)*3.0) << endl;
```

the expression **(v+2)\*3.0** evaluates to the creation of a temporary expression object and eval () invokes its interpretation.

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(59)

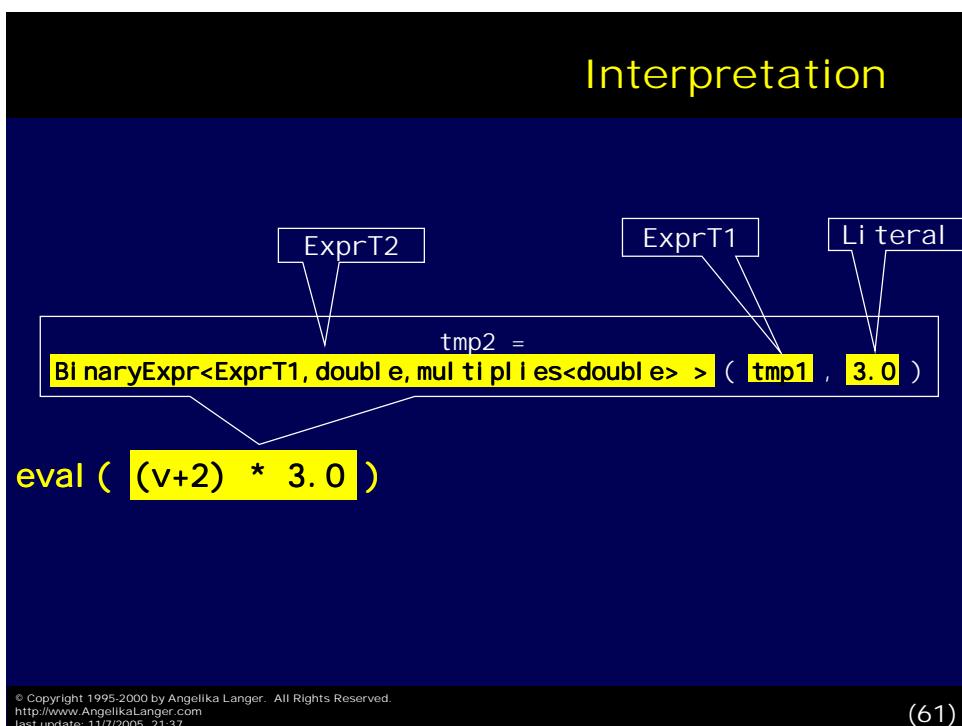
## Interpretation



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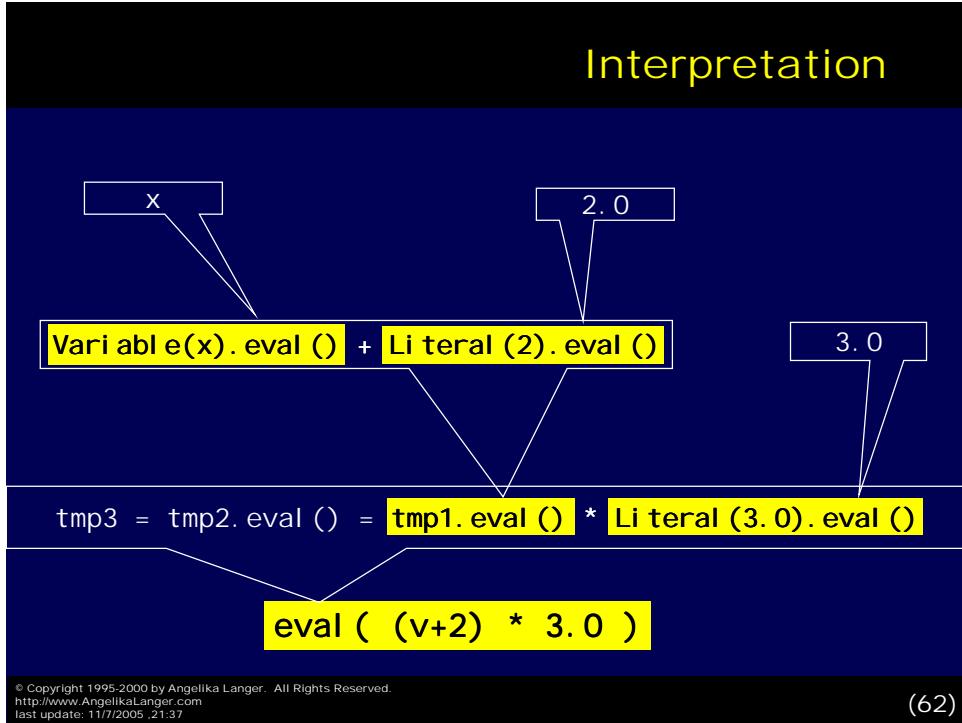
(60)

## Interpretation



(61)

## Interpretation



(62)

## Practical Applications

So far, the interpretation of the syntax tree is rather static.

- The syntax tree is created and interpreted only once.

A more dynamic usage model is possible, where a given syntax tree can be evaluated repeatedly for different input values.

```
template <class ExprT>
double integrate (ExprT e, double from, double to, size_t n)
{ double sum=0, step=(to-from)/n;
  for (double i =from+step/2; i<to; i+=step)
    sum+=e.eval (i);
  return step*sum;
}
```

```
Identity x;
cout << integrate (x/(1.0+x), 1.0, 5.0, 10) << endl;
```

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(63)

## Minor Modifications

```
class Literal {
public:   Literal (double v) : _val (v) {}
           double eval (double) const { return _val; }
private:  const double _val;
};
```

```
class Identity {
public:   double eval (double d) const { return d; }
};
```

```
template <class ExprT1, class ExprT2, class BinOp>
class BinExpr {
public:   double eval (double d) const
           { return _op(_expr1.eval (d), _expr2.eval (d)); }
...
};
```

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(64)

## Incorporating Functions

We can incorporate functions such as `sqrt()`, `exp()`, etc.:

```
template <class ExprT>
UnaryExpr<ExprT, doubl e(*)(doubl e)>
sqrt(const ExprT& e)
{return UnaryExpr<ExprT, doubl e(*)(doubl e)>(e, :: std::sqrt); }
```

and calculate the Gauss distribution:

```
doubl e si gma=2.0, mean=5.0;
const doubl e Pi = 3.141593;
cout << integrate(
1.0/(sqrt(2*Pi)*si gma) * exp(sqrt(x-mean)/(-2*si gma*si gma))
, 2.0, 10.0, 100) << endl ;
```

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(65)

## Incorporating Functions

These functions are provided via creator functions that yield unary operations:

```
template <class ExprT, class UnaryOp>
class UnaryExpr {
public:
    UnaryExpr(ExprT e, UnaryOp op=UnaryOp())
        : _expr(e), _op(op) {}
    doubl e eval(doubl e d) const { return _op(_expr.eval(d)); }
private:
    ExprTrait<ExprT>::expr_type _expr;
    UnaryOp _op;
};
```

```
template <class ExprT>
UnaryExpr<ExprT, doubl e(*)(doubl e)>
exp(const ExprT& e)
{ return UnaryExpr<ExprT, doubl e(*)(doubl e)>(e, :: std::exp); }
```

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(66)

## Further Applications in Practice

If the `interpret()` or `eval()` function is provided as an overloaded operator`()()`, the expressions can serve as function objects for the STL.

```
List<int> l;  
Identify x;  
count_if(l.begin(), l.end(), x >= 0 && x <= 100);
```

... provided the logical and comparison operators are overloaded to yield expressions that evaluate to Boolean values.

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(67)

## Agenda

- compile-time computation of constant values
  - factorial
  - square root
- compile-time evaluation of expressions
  - dot product
  - arithmetic expression
- more examples of modern template programming
  - compile-time polymorphism
  - policy classes
  - template meta-programming

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(68)

## Run-Time Polymorphism

Classic object-oriented approach:

```
class Rotatable {
public:
    virtual void rotate(int) = 0;
};

class Ellipse : public Rotatable
{
public:
    Ellipse(int x, int y)
        : Xradius(x), Yradius(y) {}
    virtual void rotate(int);
private: int Xradius, Yradius;
};
```

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(69)

## Run-Time Polymorphism

Using run-time polymorphism:

```
void vertical_flip(Rotatable& d)
{ d.rotate(180); }

Ellipse ellipse(100, 600);
vertical_flip(ellipse);

Rectangle rectangle(999, 500);
vertical_flip(rectangle);
```

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(70)

## Compile-Time Polymorphism

Replace intrusive inheritance by name commonality:

```
class Ellipse{
public: Ellipse(int x, int y)
    : Xradius(x), Yradius(y) { }
    void rotate(int);
private: int Xradius, Yradius;
};

class Rectangle {
public: Rectangle(int x, int y)
    : Xedge(x), Yedge(y) { }
    void rotate(int);
private: int Xedge, Yedge;
};
```

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(71)

## Compile-Time Polymorphism

Using compile-time polymorphism:

```
template <class Rotatable>
void vertical_flip(Rotatable& d)
{ d.rotate(180); }

Ellipse ellipse(100, 600);
vertical_flip(ellipse);

Rectangle rectangle(999, 500);
vertical_flip(rectangle);
```

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(72)

## Run-Time vs. Compile-Time Dispatch

Inheritance-based polymorphism (OOP) can be replaced by templates (GP = generic programming).

### Overhead:

- GP: code bloat & compile-time overhead
- OOP: run-time overhead

### Conformance to an interface:

- GP: common names
- OOP: a common base class

### Integration of built-in types:

- GP: overloaded operators  $\Leftrightarrow$  native operators  
function objects  $\Leftrightarrow$  function pointers
- OOP: not possible

## Agenda

- compile-time computation of constant values
  - factorial
  - square root
- compile-time evaluation of expressions
  - dot product
  - arithmetic expression
- more examples of modern template programming
  - compile-time polymorphism
  - policy classes
  - template meta-programming

## Policy Specifications

- classic quick sort function from C library
  - uses function pointer for sorting criterion

```
void qsort( void *base, size_t num, size_t width,  
           int (*compare)(const void *elem1, const void *elem2)  
 );
```

- sort algorithm from the C++ library
  - uses comparator type (function pointer or function object)

```
template<class RandomAccessIterator, class Comparator>  
void sort(RandomAccessIterator first,  
          RandomAccessIterator last,  
          Comparator cmp);
```

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(75)

## The Strategy Pattern

The Strategy pattern (also known as Policy) provides a way to configure a class or function (the *context*) with one of many behaviors (the *strategies*).

- There is a family of strategies that have a certain interface in common.
- The context uses this interface for implementing its operations.

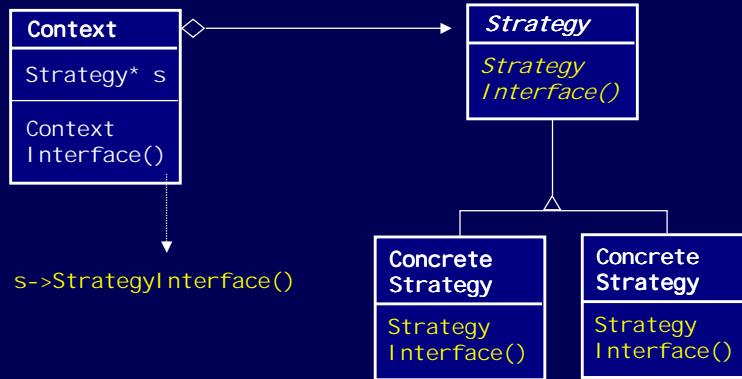
Examples:

- allocation strategies of a container
- sorting order used by a sort algorithm or a sorted collection
- synchronization policy of objects

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(77)

## The Strategy Pattern



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(78)

## An Example - Sorting of Strings

Context:

- a class or function that sorts strings, or
- a sorted collection

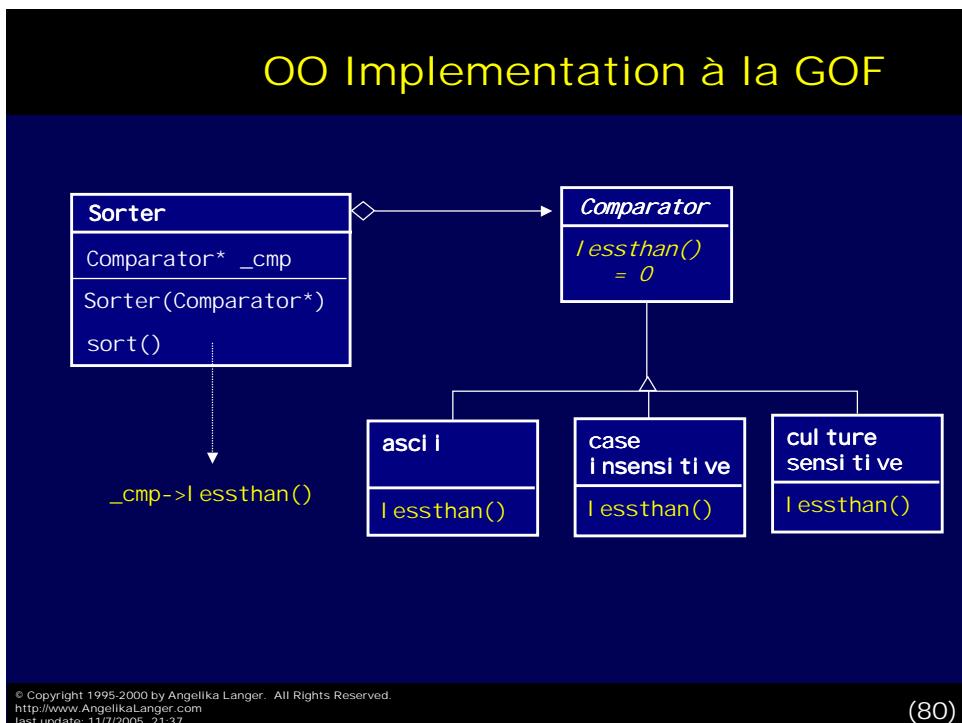
Strategy:

- ASCII sorting order
- case-insensitive sorting order
- culture-sensitive, dictionary sorting order

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(79)

## OO Implementation à la GOF



(80)

## The Context

```

class Sorter
{
public:
    explicit Sorter(Comparator* c) : _cmp(c) {}

    void sort(const StringContainer& c)
    { ... _cmp->lessThan(lhs, rhs) ... }

private:
    Comparator* _cmp;
};

```

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(81)

## The Strategies

```
class Comparator {
public:
    virtual bool
    lessThan(const string&, const string&) const = 0;
};

class Ascii : public Comparator {
public:
    virtual bool
    lessThan(const string& lhs, const string& rhs) const
    { return lhs < rhs; }
};
```

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(82)

## Using Context and Strategies

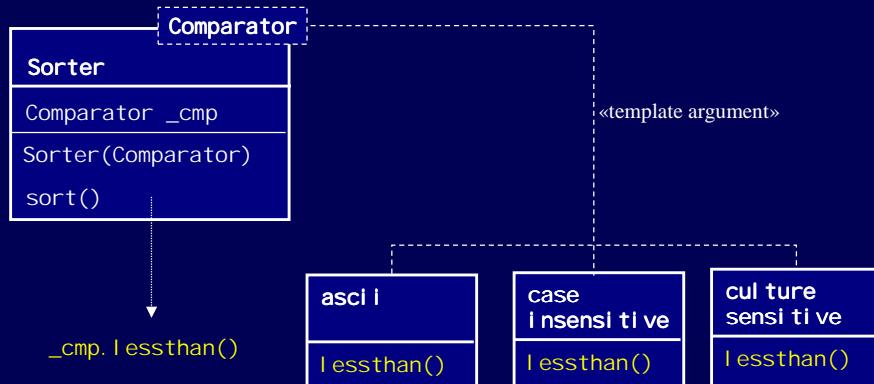
```
StringContainer cont;
Ascii      asciiCmp;
CaseInsensitiveCaseInsensitiveCmp;
Sorter    asciiSrt(&asciiCmp);
Sorter    caseInsensitiveSrt(&caseInsensitiveCmp);
asciiSrt.sort(cont);
caseInsensitiveSrt.sort(cont);
```

- cannot tell from the type of the Sorter how it sorts
- need not re-compile Sorter when new strategies are added
- run time binding for lessThan() based on type of comparator
- polymorphism through “another level of indirection”

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(83)

## Implementation Using Templates



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(84)

## The Context

```
template <typename Comparator>
class Sorter
{
public:
    explicit Sorter(const Comparator& c = Comparator())
        : _cmp(c) {}

    void sort(const StringContainer& c)
    { ... _cmp. lessThan(lhs, rhs) ... }

private:
    Comparator _cmp;
};
```

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(85)

## The Strategies

```
class Ascii {
public:
    bool lessThan(const string& lhs, const string& rhs) const
    { return lhs < rhs; }
};

class CaseInsens {
public:
    static bool lessThan(const string& lhs, const string& rhs)
    { locale loc;
        const ctype<char>& fac = use_facet<ctype<char>>(loc);
        string tmp1(lhs), tmp2(rhs);
        fac.tolower(tmp1.begin(), tmp1.end());
        fac.tolower(tmp2.begin(), tmp2.end());
        return tmp1 < tmp2;
    }
};
```

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(86)

## The Strategies

```
class CultSens {
public:
    explicit CultSens(locale l = locale()) : _loc(l) {}
    bool lessThan(const string& lhs, const string& rhs) const
    { return _loc(lhs, rhs); }
private:
    locale _loc;
};
```

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(87)

## Using Context and Strategies

```
StringContainer cont;
Sorter<Ascii> srt1;
Sorter<CaseInsens> srt2;
Sorter<CultSens> srt3(CultSens(locale("German")));

srt1.sort(cont);
srt2.sort(cont);
srt3.sort(cont);
```

- can tell from the name of the type of the Sorter how it sorts
- need to compile instantiations of Sorter for new strategies
- compile time binding for sort() and lessThan()
- polymorphism through templates

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(88)

## Evaluation of Policy Specifications

- policy specification via template parameters is more flexible
  - not restricted to just one function signature
  - both function pointers and function objects allowed
  - possibility for bundling policies

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(89)

## Examples from Standard C++

- sorting strategy and allocation policy of containers

```
template <class ElementType,
          class Comparator = Less<Key>,
          class Allocator = Allocator<Key> >
class set;
```

- character handling and allocation policy of strings

```
template<class CharacterType,
         class CharacterTraits = char_traits<charT>,
         class Allocator = Allocator<charT> >
class basic_string;
```

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(90)

## A Sorting Strategy

- comparator type `Less` from the STL
  - just one functionality provided
  - via overloaded function call operator

```
template <class T>
struct Less : binary_function<T, T, bool> {
    bool operator()(const T& x, const T& y) const;
};
```

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(91)

## A Policy Bundle

- the standard allocator
  - a bundle of functionalities
  - provided as non-static member functions

```
template <class T> class allocator {  
public:  
    ...  
  
    pointer allocate(size_t);  
    void deallocate(T* p, size_t n);  
  
    void construct(T* p, const T& val);  
    void destroy(T* p);  
};
```

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(92)

## Another Policy Bundle

- the standard character traits for type char
  - a bundle of static functionality

```
template<>  
struct char_traits<char> {  
    static bool eq(const char& c1, const char& c2);  
    static bool lt(const char& c1, const char& c2);  
  
    static char* move(char* s1, const char* s2, size_t n);  
    static char* copy(char* s1, const char* s2, size_t n);  
  
    ...  
};
```

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(93)

## Alexandrescu's Smart Pointer

- the configurable smart pointer class template from the Loki library
  - uses not just template type parameters, but also template template parameters

```
template
<
    typename T,
    template <class> class OwnershipPolicy = RefCounted,
    class ConversionPolicy = DisallowConversion,
    template <class> class CheckingPolicy = AssertCheck,
    template <class> class StoragePolicy = DefaultSPStorage
>
class SmartPointer;
```

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(94)

## Alexandrescu's Smart Pointer

- ownership policy
  - deep copy, destructive copy, no copy
  - reference counted (thread-safe or not)
- conversion policy
  - allow or disallow implicit conversion to underlying pointer type
- checking policy
  - reject null
  - no check
- storage policy
  - default storage (does delete )
  - array storage (does array delete[] )
  - heap storage (calls free())

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(95)

## Benefits of Meta-Programming

- Very good performance due to
- exclusive use of static binding;  
polymorphic behavior can be simulated statically.
  - inlining;  
enables the compiler to optimize aggressively.

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(108)

## Problems with Meta-Programming

- Debugging.  
practically not possible; there are tricks though.
- Error reporting.  
no influence on the compiler message.
- Readability of code.  
it looks awkward.
- Compilation times.  
increases by orders of magnitude.
- Compiler limits.  
truncation of identifiers, loop limits exceeded.
- Portability.  
some compilers still not support standard C++

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(109)

# References

## Expression Template Libraries

### The Blitz Project

<http://oونumerics.org/blitz/>

A C++ class library for scientific computing which uses template techniques to achieve high performance.

### PETE (Portable Expression Template Engine)

<http://www.acl.lanl.gov/pete/>

A portable C++ framework to easily add powerful expression templates.

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(110)

# References

## Expression Template Libraries

### POOMA (Parallel Object-Oriented Methods and Applications)

<http://www.acl.lanl.gov/pooma/>

- An object-oriented framework for applications in computational science requiring high-performance parallel computers.
- Considerable success in real applications including gyrokinetic particle-in-cell plasma simulation and multimaterial compressible hydrodynamics.
- Developed at Los Alamos National Laboratory, New Mexico.

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(111)

# References

## Further Links

### Research Centre Jülich

<http://www.fz-juelich.de/zam/cxx/>

An impressive directory of C++ resources such as books, articles, FAQs, other C++ pages, compilers, libraries, etc.

See in particular the links to other C++ libraries at  
<http://www.fz-juelich.de/zam/cxx/extmain.html#lib>

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## Patterns

### Design Patterns

Erich Gamma, Richard Helm, Ralph Johnson, John Vlissides  
Addison-Wesley, 1994

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